

Fast Geometric Method for Calculating Accurate Minimum Orbit Intersection Distances

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ABSTRACT

We present a new method to compute Minimum Orbit Intersection Distances (MOIDs) for arbitrary pairs of heliocentric orbits and compare it with Giovanni Gronchi's algebraic method. Our procedure is numerical and iterative, and the MOID configuration is found by geometric scanning and tuning. A basic element is the meridional plane, used for initial scanning, which contains one of the objects and is perpendicular to the orbital plane of the other. Our method also relies on an efficient tuning technique in order to zoom in on the MOID configuration, starting from the first approximation found by scanning. We work with high accuracy and take special care to avoid the risk of missing the MOID, which is inherent to our type of approach. We demonstrate that our method is both fast, reliable and flexible. It is freely available and its source Fortran code downloadable *via* our web page.

Key words: *Celestial mechanics – Minor planets, asteroids: general – Comets: general*

1. Introduction

Close encounters play important roles in several branches of Solar System science. For instance, one may consider comet dynamics involving close encounters between comets and giant planets, the study of meteor streams and meteorite delivery to the Earth involving orbital evolution into collision with our planet, and the identification of those Near Earth asteroids that are responsible for the current impact hazard. For all such purposes, attention has been paid to the problem of computing the global minimum of the Keplerian distance function referring to two confocal Keplerian orbits (*i.e.*, the distance d between a point on one orbit and a point on the other as a function of the two anomalies describing the points). This determines how close two orbiting particles may possibly come to each other in the absence of gravitational focusing, neglecting long-range perturbations.

Methods for finding and calculating the values of those global minima have been proposed in several previous papers. For instance, the situation of comets approaching giant planets was considered by Sitarski (1968) and that of meteor streams in Earth's vicinity by Babadzhanov *et al.* (1980). Close approaches between minor planets were treated by Lazović (1980, 1981). Hoots *et al.* (1984) concentrated on satellite orbits in view of the rising concern about inter-satellite collisions. Dybczyński *et al.* (1986) discussed limitations and risk of failure associated with previous methods and suggested improvements.

When the asteroid impact hazard became an issue in the 1990s, new interest also arose in the calculation of minimum approach distances to the Earth. Bowell and Muinonen (1994) thus introduced the concept of Potentially Hazardous Asteroids (PHA) with the aid of the above-described minimum distance as applied to an asteroid and the Earth. This was called the Minimum Orbit Intersection Distance (MOID), and PHA were defined as asteroids with $\text{MOID} < 0.05$ a.u. and absolute magnitude $H < 22$.

More recent results include the development of algebraic routines for calculating MOIDs or – in general – minima of the d^2 function (*e.g.*, Baluev and Kholshchevnikov 2005, Gronchi 2005). A new numerical-analytical method for MOID computation was introduced by Šegan *et al.* (2011). Currently, Gronchi's computer code as available on the web¹ is the standard tool for MOID computation. Its performance has been proved to be excellent as regards accuracy, reliability and speed. Nonetheless, we have seen a need to develop an alternative, geometric method with special advantages for use in massive computer simulations. As will be shown in a forthcoming paper (Rickman *et al.*, in preparation), one such use is for numerical calculations of average impact probabilities for comets or planet-crossing asteroids with terrestrial planets.

In this paper we provide the description of our new method of MOID calculation and its numerical implementation (Section 2) and a demonstration of its performance (Section 3). We also subject it to a critical comparison with Gronchi's method (Section 4) and briefly summarize our conclusions (Section 5). The computer source code in Fortran is freely available on the web².

2. The Method

The aim of this section is to describe in some detail all the steps leading to a calculated MOID, using a minimum of formulae and avoiding all details of computer programming. After descriptions of the preparatory phase, the meridional plane scanning and the technique of zooming in on the MOID, we present and discuss the way in which our method uses these tools in order to make sure that the proper MOID is identified and calculated with very high accuracy.

¹adams.dm.unipi.it/~gronchi/kepdist/kep_dist2.html

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2.1. Preparing the Calculations

There is no restriction on the two orbits, whose MOID is to be calculated, except for the obvious one of being confocal. Each one is defined by five orbital elements as regards the size, shape and orientation in space. With usual notations, and with reference to the ecliptic and equinox of a standard epoch, the elements of objects \mathcal{A} and \mathcal{B} may be written:

$$\begin{aligned} a_1, e_1, i_1, \omega_1, \Omega_1 & \text{ -- for body } \mathcal{A}, \\ a_2, e_2, i_2, \omega_2, \Omega_2 & \text{ -- for body } \mathcal{B}. \end{aligned}$$

Generally, neither i_1 nor i_2 equals zero, since both orbits may be inclined to the ecliptic. Our method is constructed to work only for cases, where one of the orbits is not inclined. However, this is not a limitation, because for any two orbits we may rotate the reference frame so that one of them is no longer inclined, and its perihelion direction is the reference for longitudes.

Doing this, we do not change the MOID, but the orbital elements must be recalculated. Thus, for any two orbits, the first step is to rotate the reference frame using standard equations of celestial mechanics, such that the new frame is defined by object \mathcal{A} . This results in the new set of orbital elements:

$$\begin{aligned} a_A, e_A, 0, 0, 0 & \text{ -- for body } \mathcal{A}, \\ a_B, e_B, i_B, \omega_B, \Omega_B & \text{ -- for body } \mathcal{B}, \end{aligned}$$

where $a_A = a_1$, $e_A = e_1$, $a_B = a_2$, $e_B = e_2$.

2.2. Scanning the Orbits

This tool yields a preliminary approach to the minima of the distance function. It is illustrated by Fig. 1. In this figure the orbit of object \mathcal{A} and its plane are colored blue, while the corresponding for object \mathcal{B} have orange color. Fig. 1 highlights a particular location of object \mathcal{B} . The Sun is at the origin, the reference plane is shown as the orbital plane of \mathcal{A} , and the polar axis is perpendicular to this. Now, let us consider a plane through \mathcal{B} and the poles (*i.e.*, perpendicular to the reference plane). We call this the meridional plane and indicate it in Fig. 1 as a light gray circle.

At this moment, \mathcal{A} may be situated anywhere on its orbit, but let us consider the two points, where the orbit crosses the meridional plane. Of these, we focus on the one that is close to \mathcal{B} . The Sun, \mathcal{A} and \mathcal{B} then form a triangle lying in the meridional plane.

We denote the distance between \mathcal{A} and \mathcal{B} by D_0 in this particular case, as shown in Fig. 1. Knowing the orientation of \mathcal{B} 's apsidal line, we can specify the true anomaly of \mathcal{B} , which we call ν_0 . The heliocentric distance of \mathcal{B} is thus:

$$r_{B0} = \frac{a_B(1 - e_B^2)}{1 + e_B \cos \nu_0} \quad (1)$$

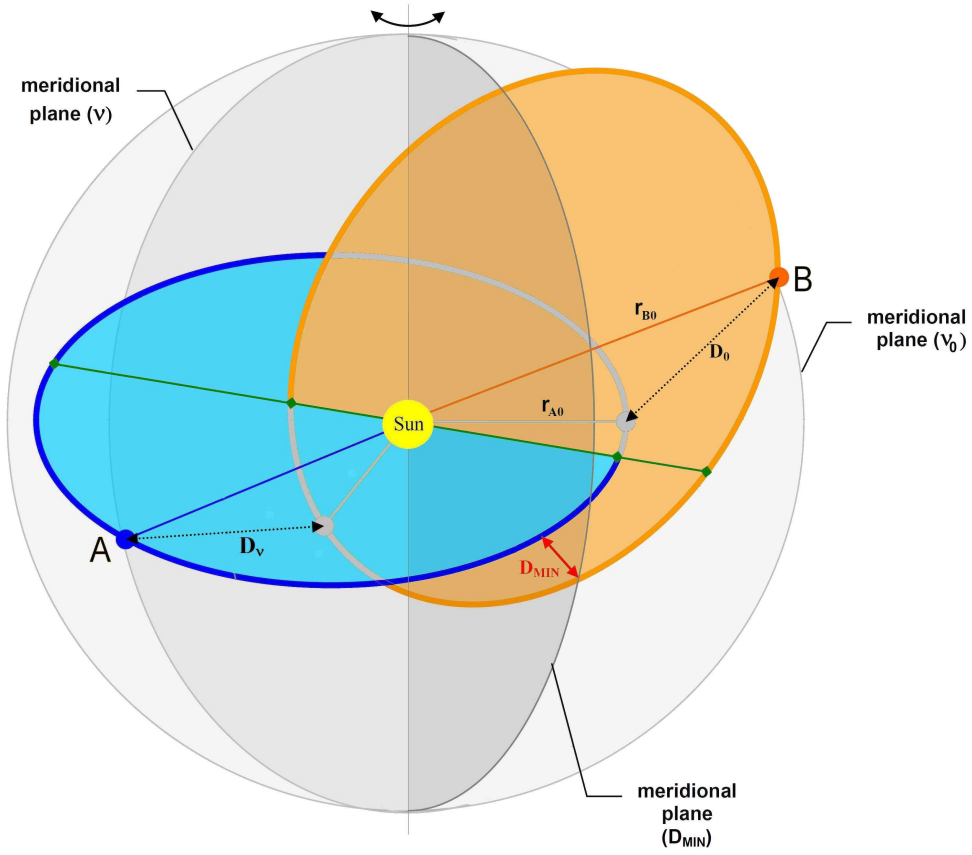


Fig. 1. Illustrative sketch of the orbital geometry of objects \mathcal{A} and \mathcal{B} . Three positions of the meridional plane are indicated along with the orbital planes of the two objects. The locations of the objects in these planes and the mutual distances are shown.

and using right-handed Cartesian coordinates as commonly defined, we obtain:

$$\begin{aligned} x_{B0} &= r_{B0} [\cos \Omega_B \cos(\omega_B + v_0) - \sin \Omega_B \sin(\omega_B + v_0) \cos i_B], \\ y_{B0} &= r_{B0} [\sin \Omega_B \cos(\omega_B + v_0) + \cos \Omega_B \sin(\omega_B + v_0) \cos i_B], \\ z_{B0} &= r_{B0} \sin(\omega_B + v_0) \sin i_B. \end{aligned} \quad (2)$$

To find the coordinates of \mathcal{A} , note that its longitude (equal to its true anomaly by the definition of the reference frame) is the same as that of object \mathcal{B} , defining the meridional plane. This is uniquely defined by the coordinates x_{B0} and y_{B0} . Calling it L_0 , and using

$$\rho_{B0} = \sqrt{x_{B0}^2 + y_{B0}^2}, \quad (3)$$

we have:

$$\begin{aligned} \cos L_0 &= x_{B0} / \rho_{B0}, \\ \sin L_0 &= y_{B0} / \rho_{B0}. \end{aligned} \quad (4)$$

We then derive the heliocentric distance of \mathcal{A} from:

$$r_{A0} = \frac{a_A(1 - e_A^2)}{1 + e_A \cos L_0}, \tag{5}$$

and as shown in Fig. 2, D_0 is given by:

$$D_0 = \sqrt{z_{B0}^2 + (\rho_{B0} - r_{A0})^2}. \tag{6}$$

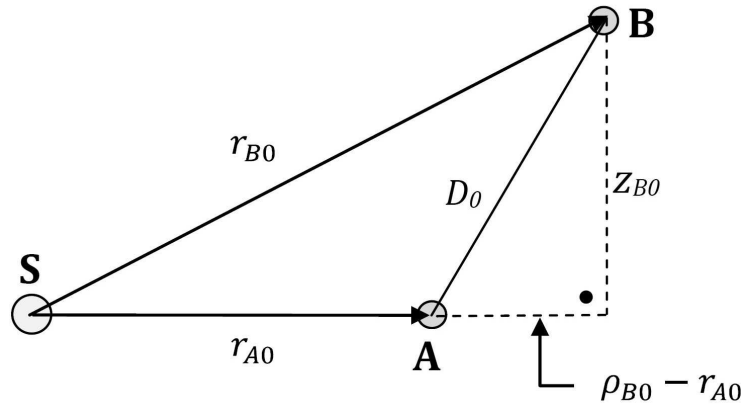


Fig. 2. The triangle formed by the Sun, \mathcal{A} and \mathcal{B} in the meridional plane defined by true anomaly v_0 .

Now, as object \mathcal{B} moves along its orbit, its true anomaly changes and the meridional plane rotates. For each position we can perform the above calculations and find a mutual, “meridional” distance D_v as a function of v . One such case, marked by the position of \mathcal{A} , is shown in Fig. 1 (medium gray meridional plane). By scanning one full revolution of \mathcal{B} , we are able to identify all local minima D_{\min} of the meridional distance, one of which is situated in the dark gray plane of Fig. 1. The criterion for detecting a local minimum is that we find a value of D_v that is smaller than both its preceding and following values in the true anomaly sequence.

The step size Δv used for the true anomaly of \mathcal{B} sets the speed of the scanning. However, there is an important issue about reliability. As has been discussed at length in previous literature, the distance function is sometimes very complicated with several critical points (maxima, minima, saddles), and our one-dimensional meridional plane scanning may thus reveal more than one minimum. Furthermore, there is a risk that it fails to detect an existing minimum that may even be the MOID. From previous work concerning the minima of the distance function – see, in particular, Gronchi *et al.* (2007) and Šegan *et al.* (2011) – we know that there may be up to four minima for an arbitrary pair of orbits, but in practice there are mostly two (sometimes one, and very rarely three or four). Thus, our scanning may return from one to four local values of D_{\min} , but we have no *a priori* guarantee

that the MOID is always to be found among those. We will return to this problem below.

In practice, we have found that a scanning step of 0.12 rad is close to optimal. This is enough to capture all the local minima of D_v and leads to a very low consumption of CPU time.

2.3. Parallel Tuning

After the scanning phase, we typically have a few values of D_{\min} , but we also have the true anomalies and full Cartesian coordinates of objects \mathcal{A} and \mathcal{B} corresponding to those distances. In Fig. 3, the positions in question are identified as A_1 and B_1 , respectively, corresponding to one of the two cases. While these positions are situated on a common meridional plane, our goal is now to move both objects separately along their orbits in order to find the smallest possible distance between them, which is no longer a meridional distance.

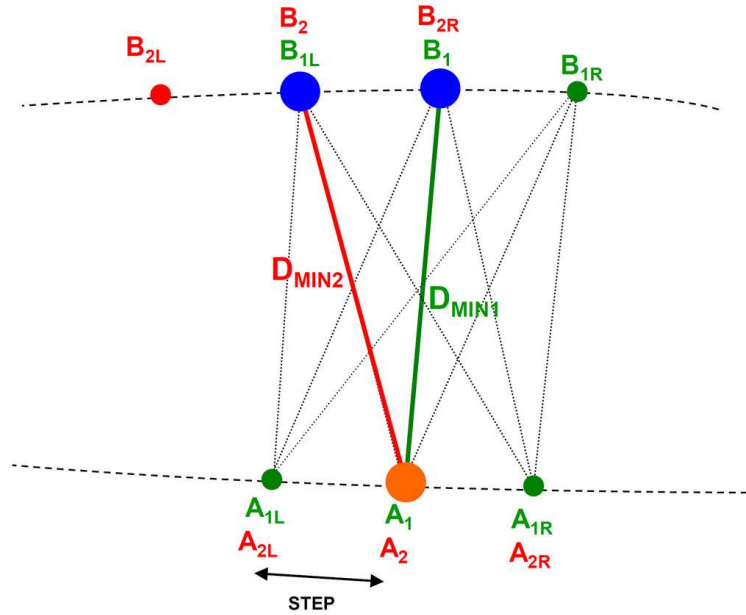


Fig. 3. Illustration of the parallel tuning method: sketch of the orbital segments (dashed curves) of objects \mathcal{A} and \mathcal{B} close to the points, where the meridional distance has a local minimum. These points are joined by a thick, green line. Varied positions of the objects are also shown, corresponding to a given step in true anomaly, and the mutual distances are shown by thin, dotted lines. The smallest of these distances is shown by the thick, red line.

The meridional distance between A_1 and B_1 is denoted $D_{\min 1}$. Now, let both \mathcal{A} and \mathcal{B} move to the right or left along their orbits by a given step Δv in true anomaly. The four resulting, varied positions are denoted A_{1R} , A_{1L} , B_{1R} and B_{1L} . Along with A_1 and B_1 , we now have six points, which lead to nine values of the distance between \mathcal{A} and \mathcal{B} . One of these ($D_{\min 1}$) has already been found, and now

we need to calculate the other eight (indicated by dotted lines in Fig. 3). For this we must use the full Cartesian coordinates of the four varied positions, whose true anomalies (L for \mathcal{A} and ν for \mathcal{B}) may take the known values for A_1 and B_1 plus or minus $\Delta\nu$. We thus proceed as follows:

$$\begin{aligned} r_A &= a_A(1 - e_A^2)/(1 + e_A \cos L), \\ r_B &= a_B(1 - e_B^2)/(1 + e_B \cos \nu). \end{aligned} \quad (7)$$

$$\begin{aligned} x_A &= r_A \cos L, \\ y_A &= r_A \sin L, \\ z_A &= 0. \end{aligned} \quad (8)$$

$$\begin{aligned} x_B &= r_B [\cos \Omega_B \cos(\omega_B + \nu) - \sin \Omega_B \sin(\omega_B + \nu) \cos i_B], \\ y_B &= r_B [\sin \Omega_B \cos(\omega_B + \nu) + \cos \Omega_B \sin(\omega_B + \nu) \cos i_B], \\ z_B &= r_B \sin(\omega_B + \nu) \sin i_B. \end{aligned} \quad (9)$$

and finally, the distance is found as:

$$D = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2}. \quad (10)$$

After all nine distances have been found, they are compared, and the smallest one is chosen. Suppose that this corresponds to segment A_1B_{1L} , as shown in Fig. 3. That segment is consequently a better approach to the MOID than the initial one, so we consider its end points (A_1 and B_{1L}) as the new reference positions, replacing A_1 and B_1 . We may call them A_2 and B_2 , respectively, and their mutual distance is $D_{\min 2}$.

We now repeat the above procedure, using the same true anomaly step. This means that we consider the six points A_{2R} , A_2 , A_{2L} , B_{2R} , B_2 and B_{2L} . For the case shown in Fig. 3, only one of these six points is new (B_{2L}), and in a general case no more than two points may be new. After calculation of the remaining Cartesian coordinates, all nine distances are again compared, and the smallest one selected. We iterate this procedure n times until we find the reference distance $D_{\min n}$ to be the smallest of the nine. In fact, we work with the distance squared when selecting the optimal pair of points, allowing some extra saving of CPU time by avoiding square root calculations.

At that point, we keep A_n and B_n as reference positions and decrease the true anomaly step by a given factor. Using thus $A_1 = A_n$ and $B_1 = B_n$, we go through the above iterations once more with the smaller step, again until we find the reference distance to be the smallest. Then we reset the system and decrease the step size again, and the procedure is repeated until the step size falls below a predefined limit. Note that the method described amounts to zooming in on the closest, local minimum of the distance function. The initial step size adopted in this procedure, which we call ‘‘parallel tuning’’, is 0.06 rad, and each decrease is achieved by applying a factor 0.15.

We have adopted two phases for the parallel tuning, as will be described in the following paragraphs. The initial tuning is the first phase, and its target step size is $5 \cdot 10^{-6}$ rad. Then follows the final tuning, which starts from the result of the initial tuning and proceeds to a final target step size, at which the procedure is terminated, of $1 \cdot 10^{-14}$ rad. Note that the mentioned parameter values for the tuning have been optimized for maximum speed while retaining perfect reliability (see Section 2.4). Only the final target step size is adapted to double precision calculations and thus independent of the optimization.

2.4. Finding the MOID

In principle, our method is to first go through the preparation step, then apply the scanning by meridional plane, and thereafter perform the initial tuning, starting from all the local meridional distance minima found during the scanning. Using the results of this initial tuning, where each closest local minimum of the distance function is determined at an accuracy of $\approx 10^{-8}$ a.u., we are almost always able to identify which one is the smallest and then zoom in on it by the final tuning. As already mentioned, the scanning finds all local minima of D_v , and if the MOID would always be close to one of these minima, the high accuracy of our parallel tuning would never fail to identify and measure the MOID.

However, there is an intrinsic problem of the meridional plane scanning method, namely, that cases exist where a local minimum of the distance function yields no corresponding minimum of the meridional distance. From the point of view of our scanning method, these are “invisible minima”. In some of these cases, the MOID may thus be invisible, and we have to find a way around this problem.

To this end, we have taken advantage of the possibility to compare our results with those computed by Gronchi’s method. We arranged the output from his code to be in double precision and could thus easily discover all cases, where our MOID differed from his. Even if a difference was very small, but still much larger than the usual difference of $< 10^{-13}$ a.u., we could positively identify a case of missed MOID in our code. Without applying any countermeasures, we found this to occur at the level of about one case in 100 000. Moreover, all of them occurred when only one minimum was detected by the scanning – in itself a rare outcome.

This fortunate circumstance led to our solution of the problem. Whenever the scanning detects only one minimum, we start the parallel tuning, not from this minimum, but from several positions of the meridional plane defined by points that are evenly distributed along the inclined orbit. Empirically, we found that at least four points need to be used in order for one of them to lead, by tuning, to the invisible minimum in case this exists. The exact choice of the starting points is not critical, but using four points we have not found any case of failure. For the sake of completeness, we need to mention one additional case of using four evenly spread starting positions. This is the extremely rare situation, when the initial tuning fails to distinguish between two nearby minima found by scanning. Note that the method

of four starting points is relatively time consuming, and thus we have to restrict it to the cases where it is really needed.

Investigating a sample of two million orbit pairs, and using the methods described above including the special treatment of the one-minimum and indistinguishable two-minima cases, we did not find a single case of discrepancy between our MOIDs and those by Gronchi. The agreement is always perfect. Thus, the only limits to the accuracy are set by the precision of the computer and, in our case, by the terminal step size of the parallel tuning, which is anyway matched to double precision calculations. However, we have to emphasize that we have no absolute guarantee against missed MOIDs with our method. We can only say that they have to be non-existent or extremely rare.

3. Checks of Performance

While the reliability and accuracy of our method thus seem to be well established, we obviously have to demonstrate that the resulting code is competitive, as compared with other existing methods, in terms of CPU time consumption.

As mentioned, we tested our accuracy by comparing our results with those generated by Gronchi's method, which is published for common use including source codes and is accessible *via* the author's web site (see above). The quality of its performance is well established through numerous applications. Thus, the finding of perfect agreement between our method and Gronchi's not only provides confidence in our results but gives credence to both methods. Such agreement has been one of our criteria when choosing the above-mentioned values of the technical parameters (step sizes) of our code, and this Section provides a few demonstrations of its speed and accuracy.

First, we define a fictitious, non-inclined "target" orbit – corresponding to the above object \mathcal{A} – and calculate the MOIDs between this object and a set of about 300 000 asteroids from the *IAU Minor Planet Center* database. Both Gronchi's method and ours are used. In Table 1 we show 20 examples of asteroid orbits of different types, and in Table 2 we present the results of the accuracy test for these orbits. In addition to the final MOID and its deviation from Gronchi's MOID as found from the double precision output, we also list the smallest values of D_{\min} found in the scanning phase and their deviations from the MOID.

For the target orbit, modeled on asteroid (21) Lutetia, we chose the following ecliptic elements (same epoch as the asteroidal elements). The inclination and longitude of ascending node were: $i = \Omega = 0$, and the rest were:

perihelion distance: $q = 2.036$ a.u.,

eccentricity: $e = 0.164$,

argument of perihelion: $\omega = 250^\circ 227$.

Table 2 illustrates the fact that our method finds MOIDs essentially equivalent to those found by Gronchi's algebraic formulae. By checking different kinds of

Table 1
Orbital elements of test asteroids

Test No.	Asteroid No.	q [a.u.]	e	i [°]	Ω [°]	ω [°]	remarks
1	1	2.55343183	0.0777898	10.58785	80.35052	72.14554	first five asteroids
2	2	2.12995319	0.2313469	34.84268	173.12520	310.03850	
3	3	1.98948966	0.2552218	12.97943	169.90317	248.22602	
4	4	2.15354370	0.0882196	7.13426	103.89537	150.08873	
5	5	2.08388391	0.1905003	5.36719	141.60955	358.80654	
6	65407	2.48391159	0.9543470	119.29902	39.00301	357.90012	orbits with largest eccentricity
7	20461	2.36382356	0.9006860	160.41316	297.34820	102.45000	
8	3200	0.13964163	0.8901393	22.23224	265.28749	322.11933	
9	2212	0.35420623	0.8363753	11.68912	28.13011	208.66724	
10	4197	0.52469070	0.7715449	12.56792	7.25167	122.30952	
11	P5447	2.74144856	0.1153501	0.00431	272.90217	251.43828	orbits with smallest inclination
12	U9154	2.50571901	0.1924270	0.01522	94.14405	304.71343	
13	53910	2.11312640	0.1215091	0.02244	321.26045	109.96758	
14	G5525	2.09876663	0.1543590	0.02731	88.64817	67.91991	
15	R4450	2.67112178	0.1328536	0.02809	41.39822	274.65080	
16	61395	1.99601821	0.1875129	1.26622	238.06043	31.32645	orbits with smallest MOID
17	64112	2.03086844	0.1653922	0.66023	339.21518	89.47548	
18	27710	1.77550824	0.1928808	3.43901	140.55651	216.20834	
19	61096	1.96745453	0.1837814	3.69269	98.95749	227.52626	
20	56127	2.15731280	0.1007470	2.91058	138.77805	231.93187	

orbits, we also have a good indication that in practice, there is no special kind, for which the agreement is less than perfect. Low inclination orbits or orbits with very small MOIDs are no exceptions.

Moreover, even if we would limit ourselves to the scanning phase and skip the tuning, the resulting, smallest D_{\min} values are not far from the correct results. Of course, they are somewhat larger than the MOIDs, and sometimes they may miss the MOID entirely, though such cases are rare and do not appear in Table 2. Moreover, we have found that all MOIDs that are missed by the scanning have values very close to the “apparent” MOIDs actually detected, meaning that D_{\min} is still a fair approximation.

There are two key features that characterize our method and make it competitive: (1) the meridional plane scanning that identifies the minima efficiently; and (2) the parallel tuning that allows to quickly zoom in on the MOID. Together, they allow us to perform the MOID calculations reliably, accurately and with a low consumption of CPU time.

Table 2
Results of MOID calculations for the test asteroids

Test No.	Asteroid No.	remarks	MOID [a.u.] <i>W-R</i>	Δ_{14}	D_{\min} [a.u.] <i>W-R</i>	Δ_3
1	1		0.13455874348909	0	0.1346719	0
2	2	first	0.00289925623680	0	0.0151316	12
3	3	five	0.07817951779390	0	0.0784614	0
4	4	asteroids	0.08735595371552	0	0.0878071	0
5	5		0.14532630925408	0	0.1455354	0
6	65407		0.26938418933051	0	0.2709789	2
7	20461	orbits with	0.54491059333263	0	0.5479893	3
8	3200	largest	0.70855959609279	-48	0.7152397	7
9	2212	eccentricity	0.03943927946198	0	0.1120682	73
10	4197		0.18225709092897	0	0.2183000	36
11	P5447		0.14766834758223	0	0.1483890	1
12	U9154	orbits with	0.00010493251317	0	0.0102467	10
13	53910	smallest	0.00030783183432	0	0.0141584	14
14	G5525	inclination	0.00098583168214	0	0.0084652	7
15	R4450		0.20707625146740	0	0.2087313	2
16	61395		0.00000003815330	0	0.0069443	7
17	64112	orbits with	0.00000419348257	0	0.0011561	1
18	27710	smallest	0.00000627704688	0	0.0402762	40
19	61096	MOID	0.00000785853673	0	0.0043333	4
20	56127		0.00001189165231	0	0.0100350	10

W-R denotes values found by our method, and Δ_{14} and Δ_3 denote the deviations of these values of MOID and D_{\min} from Gronchi's MOID in units of 10^{-14} and 10^{-3} a.u., respectively.

As a practical test of computing speed, we have chosen five real asteroids as object \mathcal{A} ("target object") and computed their MOIDs with respect to all other asteroids from the MPC database as object \mathcal{B} ("projectile object"). For the targets, our selection was *ad hoc* except for a preference for space mission targets, and the projectiles were limited to absolute magnitudes $H \leq 16.5$ in order to be at least km-sized. Fig. 4 shows a log-log plot of the cumulative distributions of MOIDs for the five targets, excluding all MOIDs larger than 1 a.u. Each curve is based on more than 280 000 MOIDs and needs less than one minute of CPU time on a fast personal computer.

The curve for (25143) Itokawa stands out by very small numbers for the smallest MOIDs, because its orbit is mainly situated interior to the main belt ($a = 1.32$ a.u.). To a lesser extent, the curve for (318) Magdalena shows the same kind

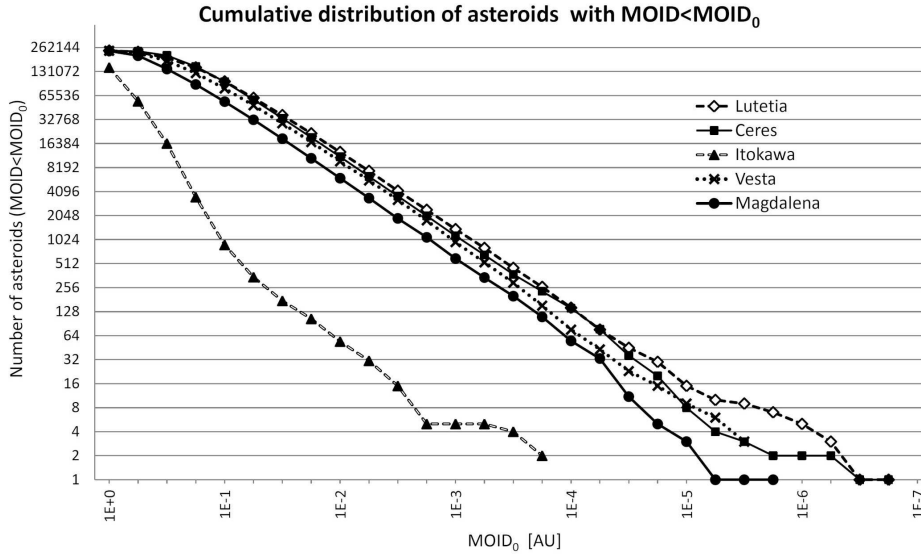


Fig. 4. Cumulative MOID distributions: The number of MOIDs less than a given value is plotted vs. that value, considering all km-sized asteroids in the MPC database.

of deficiency, being situated mostly exterior to the belt ($a = 3.19$ a.u.). Note that for the largest asteroids (Ceres, Vesta and Lutetia) there may actually be projectiles with MOIDs smaller than the target radius. Of course, the situation changes continually if one considers perturbed orbits, but we may statistically conclude that at any time, there is a non-zero number of km-sized main belt asteroids among the currently known ones on such, potentially collisional tracks.

We devised one extra check on the accuracy of our MOIDs by calculating them in cases where objects \mathcal{A} and \mathcal{B} are the same. Of course, the result in such a case must be that the MOID equals zero. In the limited number of tests that we made, this was always verified.

4. Critical Comparison of MOID Calculation Tools

The main difference between the two MOID calculation tools under consideration – ours and Gronchi’s – is that our method is numerical based on geometry, while the latter is a computer coding of algebraic formulae. However, we now need to compare their performance and quality of results.

Let us first describe the disadvantages of our method. While Gronchi’s algebraic method yields a complete solution, where all minima, maxima, and saddles are accurately found for each pair of orbits, our method only yields the MOID. It would not be difficult to find other critical points with our method, and indeed there are special situations where this would be very useful, but it was not our goal. A more important disadvantage of our method is the problem with missing MOIDs (Sections 2 and 3). We have avoided this problem to the extent that we do not detect

any missed MOIDs in millions of tests, but one should keep in mind that the risk of a miss (*i.e.*, calculating the wrong minimum) cannot vanish entirely.

Turning to advantages, we first note that our method does not have fixed accuracy and speed, because our downloadable code can easily be modified with respect to the values of the technical parameters. This may be an important advantage because, depending on the purpose, one may trade accuracy or reliability against speed. For single MOIDs, of course the speed does not matter, but in cases of millions or more calculations (*e.g.*, for mapping purposes) precision may be sacrificed and speed may be gained by increasing the step sizes. Moreover, the code may be further modified so as to avoid the safety measure of tuning from four positions of the meridional plane, which is relatively time consuming, at the price of running a risk to miss the MOID in a few cases.

The accuracy of the open access computer code at Gronchi's web site is 10^{-7} a.u. as given by the number of decimals provided. To find the intrinsic accuracy, we had to modify the output commands. Our tests on millions of orbit pairs then revealed that the two methods generally yield the same results to within 10^{-14} a.u.

To compare the computing speed between our code and Gronchi's, we used the Fortran source codes from Gronchi's web page and removed all commands that could slow down the calculations (*e.g.*, screen commands). Then we compiled and ran both programs in the same computer environment, calculating 100 000 MOIDs between (21) Lutetia and other main belt asteroids. The results were found in about 9 s to 15 s with our method *vs.* 40 s, when using Gronchi's method. The reason for the variation when using our code is that the likelihood of having to use the safety measure against missing MOIDs varies with the orbits considered. If we would turn this feature off, the time consumption would always be less than nine seconds.

One interesting advantage of our method is that we can obtain temporary results. When calculating a MOID, we have a first result already after the scanning by using the smallest value of D_{\min} . This result usually differs by 10^{-2} a.u. or less from the accurate value. If we look for MOIDs less than some threshold value, for instance looking for possible impacts, we may thus rely on temporary results and, using some margin, exclude numerous orbits from the rest of the calculation by noting that they have no chance to satisfy the applied criterion. This may result in significant, additional savings of CPU time.

A summary of the above, critical comparison is presented in Table 3. But in addition to this, let us mention a final virtue of our method that offers interesting possibilities. This comes from the fact that we always investigate the vicinity of the MOID points on the two orbits during the calculation. If the MOID is so small that potential collisions may occur, we are then able not only to establish this fact, but also to scan and measure the extent of the surrounding zone allowing collisions in order to estimate the impact probability. This feature will be explored and used in a forthcoming paper (Rickman *et al.*, in preparation).

T a b l e 3
Selected features of two MOID calculation methods

Gronchi's method	Our method
algebraic method	numerical/iterative method
fixed accuracy and speed	interchangeable accuracy and speed
test speed on standard CPU core: 40 s per 100 000 MOIDs	test speed on standard CPU core: 9–15 s per 100 000 MOIDs
estimated accuracy: $\sim 10^{-7}$ a.u. (web page) 10^{-14} a.u. (intrinsic)	accuracy: 10^{-14} a.u.
always catches the MOID	may miss the MOID, but risk $< 10^{-6}$
temporary results: impossible	temporary results: possible, allowing to speed up calculations
gives all critical points, without additional time consumption	may obtain other critical points, but with additional, slight time consumption

5. Conclusions

Let us finally note that, even though our geometric scanning method may appear unrelated to recent analytic work on MOID computation, including Gronchi's method, it does rely on and make use of existing knowledge. As we have shown in this paper, it is definitely competitive in terms of speed and accuracy and is moreover flexible. In particular, the user may trade one for the other by simply setting the values of a few technical parameters (step sizes). There is no proof that the scheme that we use (in particular, using the meridional plane) is the best among all possible schemes, but it is not likely that any major improvement could be made.

In terms of practical use in Solar System dynamics, our method will certainly have its major advantages in work involving massive simulations. If only one special MOID is looked for, there is nothing that favors either our method or Gronchi's, but the availability of both makes it possible to obtain a useful check. The real benefit of our method, on the other hand, is likely to occur in situations where millions or billions of MOIDs are wanted, and especially if information about the vicinity of the MOID configuration is also of interest.

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